

The conformal gauge to the derivative gauge for worldsheet gravity

Sudhaker Upadhyay*

Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

The BRST quantizations of worldsheet gravity corresponding to final more acceptable derivative gauge and the standard conformal gauge are studied. We establish a mapping between these two gauges utilizing FFBRST formulation in standard way. Therefore, we are able to declare that the problems associated with Virasoro constraints are the gauge artifact.

I. INTRODUCTION

It has been found that the BRST formalism is helpful in deriving the full spectrum of low-dimensional string and W-string theories [1, 2]. For instance, in the handling of anomalies in world-sheet chiral algebras the appropriateness of the BRST formalism gives full control. In case of the non-critical bosonic string, the presence of a propagating Liouville mode which is originated by the worldsheet anomaly makes the worldsheet gravity non-trivial. A worldsheet W_3 gravity described by an A_2 Toda theory is produced by anomalies in the W_3 string [3]. The anomalous Ward identities description for nonlinear chiral worldsheet algebras such as W_3 is made more difficult by the complexity and off-diagonal nature of the anomalies. The approach made in [4–6] to the W_N gravity case ran into the difficulty that a consistent set of conditions to impose on the background gauge fields to eliminate the anomalies could not be derived owing to their off-diagonal structure was extended in [7]. These difficulties were actually related to our incomplete knowledge of W_3 geometry. Further, a reformulation of the BRST quantization procedure for worldsheet gravity and the derivation of anomalous Ward identities were made in [8] which are useful for understanding the dynamics of non-critical worldsheet gravity.

On the other hand, the BRST formalism has proven to be the most powerful approach to the quantization of string/gauge theories. The generalization of BRST symmetry, known as finite field-dependent BRST (FFBRST) transformation, has been studied firstly in [9]. Further it has been found enormous applications in the diverse gauge theories [9–26]. For instance, more recently, the gauge-fixing and ghost terms corresponding to Landau and maximal Abelian gauge have been produced for the Cho-Faddeev-Niemi decomposed $SU(2)$ theory using FFBRST transformation [19]. However, the connection between linear and non-linear gauges for perturbative quantum gravity at both classical and quantum level has been established utilizing FFBRST transformation [20]. In another problem, the quantum gauge freedom studied by gaugeon formalism has also been addressed for quantum gravity [21] as well as for Higgs model [22]. The FFBRST transformations have been employed for the lattice gauge theory [25] and the relativistic point particle model [24]. Recently, the such transformation is studied in relatively different manner in [27, 28]. However, such formulation has not been discussed so-far for the worldsheet gravity. This gives us a glaring omission to study such transformation in the context of Virasoro gravity theory where one needs to fix gauge twice.

In this work, we first develop the methodology for FFBRST transformation for the gravity theory as a gauge theory. In this context we compute the finite Jacobian for the functional measure which depends on the finite field-dependent parameter implicitly. Such Jacobian actually modifies the effective action of the theory. We discuss the BRST quantization Virasoro worldsheet gravity from the different gauge perspectives. In this scenario we found that the derivative gauge is actually more acceptable than the standard conventional gauge. Further, we generalize the BRST transformation corresponding to the conventional gauge by making the infinitesimal parameter finite and field dependent. Further, we construct an specific parameter such that the Jacobian corresponding to the path integral measure takes

*Electronic address: sudhakerupadhyay@gmail.com; sudhaker@iitk.ac.in

the theory from the conventional gauge to the derivative gauge for worldsheet gravity. Since the problems associated with Virasoro constraints appear only in conventional gauge but not in the derivative gauge [8]. Therefore, we overcome this difficulty by connecting the conventional gauge to the derivative gauge.

We organize this paper in following way. In section II, we provide the details of FFBRST mechanism. In section III, we sketch briefly the BRST quantization for Virasoro gravity with the help of two examples. In section IV, we derive FFBRST transformation for such gravity theory to establish the connection between the conventional and derivative gauge. In the last section we summarize the results.

II. FFBRST TRANSFORMATION: METHODOLOGY

To analyse the FFBRST transformation, we start with the usual BRST transformation for the (generic) fields ϕ written compactly as

$$\delta_b \phi = s_b \phi \eta, \quad (1)$$

where $s_b \phi$ is the BRST (Slavnov) variation of the fields and η is an infinitesimal, anticommuting and global parameter. Such transformation is nilpotent in nature, i.e. $\delta_b^2 = 0$, with and/or without use of equation of motion of the antighost fields called as on-shell and/or off-shell nilpotent respectively. It may be observed that to be symmetry of Faddeev-Popov effective action it is not necessary to η to be infinitesimal and field-independent as long as it does not depend on the space-time explicitly. In fact the following finite field-dependent BRST (FFBRST) transformation has been introduced which preserves the same form as the BRST transformation

$$\delta_b \phi = s_b \phi \Theta[\phi], \quad (2)$$

except the field-dependent parameter $\Theta[\phi]$ which does not depend on spacetime.

Now, we briefly sketch the necessary steps to construct the FFBRST transformation. The first step is to make all the fields ϕ , a parameter ($\kappa : 0 \leq \kappa \leq 1$) dependent by continuous interpolation in such a way that fields $\phi(x, \kappa = 0) = \phi(x)$ are the initial fields and $\phi(x, \kappa = 1) = \phi'(x)$ are the transformed fields. Furthermore, the infinitesimal parameter η is made field dependent which characterizes the following infinitesimal field-dependent BRST transformation:

$$\frac{d}{d\kappa} \phi(x, \kappa) = s_b \phi(x, \kappa) \Theta'_b[\phi(x, \kappa)]. \quad (3)$$

Here Θ' denotes the infinitesimal field-dependent parameter. The integration of such transformation from $\kappa = 0$ to $\kappa = 1$ leads to the following FFBRST transformations [9]

$$\delta_b \phi(x) = \phi'(x) - \phi(x) = s_b \phi(x) \Theta[\phi], \quad (4)$$

where $\Theta[\phi]$ is (an arbitrary) finite field dependent parameter. The parameters $\Theta[\phi]$ and $\Theta'[\phi]$ are related by [9]

$$\Theta[\phi(x, \kappa)] = \Theta'[\phi(x)] \frac{\exp f[\phi(x)] - 1}{f[\phi(x)]}, \quad (5)$$

where the functional $f[\phi]$ is given by

$$f[\phi] = \sum_i \frac{\delta \Theta'(x)}{\delta \phi_i(x)} s_b \phi_i(x). \quad (6)$$

The resulting FFBRST transformation leaves the Faddeev-Popov effective action invariant. However the path integral measure defined by $(\mathcal{D}\phi)$ and therefore the generating (vacuum to vacuum) functional defined by

$$Z[0] = \int [\mathcal{D}\phi] e^{iI}, \quad (7)$$

get changed non-trivially under such FFBRST transformation. Therefore, the Jacobian is responsible for these changes. Now to compute the Jacobian for path integral measure we first write

$$\mathcal{D}\phi = J[\phi(\kappa)]\mathcal{D}\phi(\kappa). \quad (8)$$

We know that this non-trivial Jacobian can be replaced (within the functional integral) by the local polynomial as [9]

$$J[\phi(\kappa)] \rightarrow e^{iS_1[\phi(\kappa)]}, \quad (9)$$

where $S_1[\phi(\kappa)]$ is the local functional of fields $\phi(x)$, iff the following condition gets satisfy:

$$\int [\mathcal{D}\phi] \left[\frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1}{d\kappa} \right] \exp i[I + S_1] = 0, \quad (10)$$

where the change in Jacobian has the following explicit expression:

$$\frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} = - \int d^2z \sum_{\phi} \left[\pm s_b \phi \frac{\delta \Theta'_b[\phi(\kappa)]}{\delta \phi(\kappa)} \right]. \quad (11)$$

Consequently under such process our original generating functional modifies as follows:

$$\int [\mathcal{D}\phi] e^{iI[\phi]} \xrightarrow{\text{FFBRST}} \int J[\phi] [\mathcal{D}\phi] e^{i(I[\phi])} = \int [\mathcal{D}\phi] e^{i(I[\phi] + S_1[\phi])}. \quad (12)$$

Here $S_1[\phi]$ is not an arbitrary functional rather it depends on the choice of finite field-dependent parameter. Therefore, the two different effective actions can be related through FFBRST transformation with appropriate choices of finite parameter.

III. BRST QUANTISATION OF VIRASORO (W_3) GRAVITY

In this section, we analyse the theory in conventional conformal gauge and the derivative gauge and their importance.

A. Conventional BRST quantization

Similar to the bosonic string, that undergoes a preliminary stage of gauge fixing that includes the condition in complex light-cone variables z, \bar{z} of type

$$\gamma_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & h \end{pmatrix}, \quad (13)$$

the chiral Virasoro gravity action in the preliminary gauge is defined by

$$I_1 = \frac{1}{\pi} \int d^2z \left(-\frac{1}{2} \bar{\partial} \varphi^i \partial \varphi^i + \frac{1}{2} h \partial \varphi^i \partial \varphi^i \right), \quad (14)$$

where $\varphi^i (i = 0, 1, \dots, D-1)$, refers a set of matter fields and h denotes the remaining unfixed component of the two-dimensional metric. The action (14) is invariant under the following gauge transformation:

$$\begin{aligned} \delta \varphi^i &= \varepsilon \partial \varphi^i, \\ \delta h &= \bar{\partial} \varepsilon + \varepsilon \partial h - \partial \varepsilon h, \end{aligned} \quad (15)$$

where ε is a bosonic parameter of transformation. To remove the redundancy in gauge degrees of freedom due to gauge symmetry we choose the the final conventional conformal gauge condition $h = h_{back}$. Incorporating this at quantum level we get the following action:

$$I_1 = \frac{1}{\pi} \int d^2z \left(-\frac{1}{2} \bar{\partial} \varphi^i \partial \varphi^i - b \bar{\partial} c + \pi_h (h - h_{back}) - h(T_{mat} + T_{gh}) \right). \quad (16)$$

Here π_h is an auxiliary field and b, c are Faddeev-Popov ghost fields. T_{mat} and T_{gh} are the energy-momentum tensors for the matter fields and ghost fields respectively, having following expressions:

$$\begin{aligned} T_{mat} &= -\frac{1}{2} \partial \varphi^i \partial \varphi^i, \\ T_{gh} &= -2b \partial c - \partial b c. \end{aligned} \quad (17)$$

Now the effective action (16) respects the following BRST symmetry:

$$\begin{aligned} \delta_b \varphi^i &= -c \partial \varphi^i \eta, \\ \delta_b h &= -(\bar{\partial} c + c \partial h - \partial c h) \eta, \\ \delta_b c &= c \partial c \eta, \\ \delta_b b &= \pi_h \eta, \\ \delta_b \pi_h &= 0, \end{aligned} \quad (18)$$

where η denotes the anticommuting global parameter. The physical state can be spanned by restricting it with the help of Noether's charge $Q = \int d^2z (T_{mat} + \frac{1}{2} T_{gh})$ as follows $Q|phys\rangle = 0$.

B. Derivative gauge BRST quantization

In this subsection, we fix the final gauge of Virasoro gravity by choosing the derivative gauge condition $\bar{\partial} h = 0$ rather than the conventional gauge. For this gauge choice the action in the preliminary gauge (14) gets the following expression:

$$I_2 = \frac{1}{\pi} \int d^2z \left(-\frac{1}{2} \bar{\partial} \varphi^i \partial \varphi^i - h T_{mat} + \pi_h \bar{\partial} h - b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h) \right). \quad (19)$$

Here we note that due to the derivative gauge condition, the ghost action becomes second order in $\bar{\partial}$ derivatives. To use the canonical formalism, we need to introduce auxiliary fields in order to put the ghost sector into first-order form. Therefore, we define conjugate momenta corresponding to the fields c and b ,

$$\begin{aligned} \pi_c &= -\bar{\partial} b, \\ \pi_b &= \bar{\partial} c + c \partial h - \partial c h. \end{aligned} \quad (20)$$

With the help of these momenta the second-order action (19) can be written in first-order form as

$$I_2 = \frac{1}{\pi} \int d^2z \left(-\frac{1}{2} \bar{\partial} \varphi^i \partial \varphi^i + \pi_h \bar{\partial} h - \pi_b \bar{\partial} b - \pi_c \bar{\partial} c - \pi_b \pi_c - h(T_{mat} + T_{gh}) \right), \quad (21)$$

where the expression of T_{gh} is given by

$$T_{gh} = -2\pi_c \partial c - \partial \pi_c c. \quad (22)$$

The effective action (21) remains invariant under following BRST transformations:

$$\delta_b \varphi^i = -c \partial \varphi^i \eta,$$

$$\begin{aligned}
\delta_b h &= -\pi_b \eta, \\
\delta_b c &= c \partial c \eta, \\
\delta_b \pi_c &= (T_{mat} + T_{gh}) \eta, \\
\delta_b b &= \pi_h \eta, \\
\delta_b \pi_b &= 0, \\
\delta_b \pi_h &= 0.
\end{aligned} \tag{23}$$

Here these transformations are now canonical. The conserved charge corresponding to such symmetry is calculated using Noether's theorem as

$$Q = \int dz \left(c(T_{mat} + \frac{1}{2}T_{gh}) + \pi_h \pi_b \right). \tag{24}$$

This charge helps in constructing the physical state from total Hilbert space. The consequence of derivative gauge is a considerable simplification of the BRST formulation, the evaluation of anomalies and the expression of Wess-Zumino consistency conditions (see for details [8]).

IV. FFBRST TRANSFORMATION FOR VIRASORO GRAVITY

In this section we generalize the BRST transformation (18) to show that the derivative gauge can naturally be derived by operating FFBRST operator on generating functional corresponding to conventional gauge. In this context, the FFBRST transformation is constructed by

$$\begin{aligned}
\delta_b \varphi^i &= -c \partial \varphi^i \Theta[\phi], \\
\delta_b h &= -(\bar{\partial} c + c \partial h - \partial c h) \Theta[\phi], \\
\delta_b c &= c \partial c \Theta[\phi], \\
\delta_b b &= \pi_h \Theta[\phi], \\
\delta_b \pi_h &= 0,
\end{aligned} \tag{25}$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter. For different choices of such parameter one may produce different scenario. For instance, we compute the finite parameter obtainable from the following infinitesimal parameter:

$$\Theta'[\phi] = -\frac{1}{\pi} \int d^2 z b (h - h_{back} - \bar{\partial} h). \tag{26}$$

Exploiting the expression (11) we calculate the infinitesimal change in Jacobian as follows

$$\frac{1}{J} \frac{dJ}{d\kappa} = \frac{1}{\pi} \int d^2 z [b \bar{\partial} c - \pi_h (h - h_{back}) - h(2b \partial c + \partial b c) + \pi_h \bar{\partial} h - b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h)]. \tag{27}$$

Now, to evaluate the finite Jacobian we choose the following expression for local functional S_1 as discussed in condition (9):

$$\begin{aligned}
S_1[\phi, \kappa] &= \int d^2 z [\xi_1(\kappa) b \bar{\partial} c + \xi_2(\kappa) \pi_h (h - h_{back}) + \xi_3(\kappa) h T_{gh} \\
&\quad + \xi_4(\kappa) \pi_h \bar{\partial} h + \xi_5(\kappa) b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h)].
\end{aligned} \tag{28}$$

The choices for constant parameters $\xi_i(\kappa)$, $i = 1, 2, \dots, 5$ are made in such a way that these must vanish at $\kappa = 0$. The condition (10) in tandem with (27) and (28) leads

$$\begin{aligned}
\frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1}{d\kappa} &= \int d^2 z \left[(\xi'_1 - \frac{1}{\pi}) b \bar{\partial} c + (\xi'_2 + \frac{1}{\pi}) \pi_h (h - h_{back}) + (\xi'_3 - \frac{1}{\pi}) h T_{gh} \right. \\
&\quad \left. + (\xi'_4 - \frac{1}{\pi}) \pi_h \bar{\partial} h + (\xi'_5 + \frac{1}{\pi}) b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h) \right] = 0.
\end{aligned} \tag{29}$$

Equating the coefficients of each terms of the above from LHS to RHS, we get the following (exactly solvable) first-order differentiable equations:

$$\xi'_1 - \frac{1}{\pi} = 0, \quad \xi'_2 + \frac{1}{\pi} = 0, \quad \xi'_3 - \frac{1}{\pi} = 0, \quad \xi'_4 - \frac{1}{\pi} = 0, \quad \xi'_5 + \frac{1}{\pi} = 0. \quad (30)$$

The solutions for the above equations are

$$\xi'_1 = \frac{1}{\pi}\kappa, \quad \xi'_2 = -\frac{1}{\pi}\kappa, \quad \xi'_3 = \frac{1}{\pi}\kappa, \quad \xi'_4 = \frac{1}{\pi}\kappa, \quad \xi'_5 = -\frac{1}{\pi}\kappa. \quad (31)$$

Plugging these identifications to (28) we get the exact expression for $S_1[\phi, \kappa]$ as

$$\begin{aligned} S_1[\phi, \kappa] = & \frac{1}{\pi} \int d^2z \left[\kappa b \bar{\partial} c - \kappa \pi_h (h - h_{back}) + \kappa h T_{gh} \right. \\ & \left. + \kappa \pi_h \bar{\partial} h - \kappa b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h) \right], \end{aligned} \quad (32)$$

which vanishes at $\kappa = 0$, however, at κ it contributes to calculate the finite Jacobian as follows

$$\begin{aligned} J = e^{iS_1[\phi, 1]} = & \exp \left[\frac{i}{\pi} \int d^2z \left[b \bar{\partial} c - \pi_h (h - h_{back}) + h T_{gh} \right. \right. \\ & \left. \left. + \pi_h \bar{\partial} h - b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h) \right] \right]. \end{aligned} \quad (33)$$

With this Jacobian our original generating functional changes as follows

$$\int [\mathcal{D}\phi] e^{iI_1[\phi]} \xrightarrow{FFBRST} \int [\mathcal{D}\phi] e^{i(I_1[\phi] + S_1[\phi])}, \quad (34)$$

where

$$\begin{aligned} I_1 + S_1[\phi, 1] = & \frac{1}{\pi} \int d^2z \left(-\frac{1}{2} \bar{\partial} \varphi^i \partial \varphi^i - h T_{mat} + \pi_h \bar{\partial} h - b \bar{\partial} (\bar{\partial} c + c \partial h - \partial c h) \right), \\ = & I_2[\phi], \end{aligned} \quad (35)$$

which is an effective action for the derivative gauge. We may note that the Virasoro constraints (putting by hand) come in the picture only in conventional gauge (see [8] for details). However, it is shown there that all the problems associated with the Virasoro constraints get resolved naturally in the derivative gauge case. It means that these problems depend on the choice of gauges and hence are the gauge artifact. Remarkably, using standard FFBRST transformation one can switch the theory from the standard conformal gauge to the derivative gauge which is more acceptable also in the sense that the evaluation of anomalies and the expression of Wess-Zumino consistency conditions.

V. CONCLUSION

The BRST quantization procedure for chiral worldsheet gravity by the adoption of a derivative gauge condition, and the introduction of momenta in order to put the ghost sector of the theory back into first-order form, are well studied in [8]. In the derivative gauge the BRST formalism for worldsheet gravity produces the formalism canonical in the sense that the BRST transformations of all fields now arise as canonical transformations generated by the BRST charge [8].

In this paper we have provided the basic mechanism of the FFBRST transformation. Further, we have discussed the Virasoro gravity from the BRST perspective by considering the standard conventional (conformal) gauge and the derivative gauge. The derivative gauge has found more important to deal with such theory because in making of the standard conformal gauge one loses the Virasoro constraints as field equations. We have generalized the BRST transformation to obtain the FFBRST transformation

corresponding to the conventional gauge. Notably, we have found that the derivative gauge-fixed action (which is more acceptable) can be obtained naturally (within the functional integral) by operating the FFBRST transformation on the generating functional for the Virasoro gravity corresponding to the conventional gauge. We have shown this result explicitly by calculation. So this inspection allows one to perform the analysis the theory in conventional gauge where the ghost sector are in first-order, however, wherever it finds difficulty in this gauge one can switch the theory in derivative gauge by applying FFBRST transformation. It will be interesting to study the worldsheet gravity in the Batalin-Vilkovisky formulation as there anomalies are present. Our analysis might be helpful in the complete understating to W_3 gravity.

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